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EIGHTH EDITION

BIRD'S BASIC ENGINEERING MATHEMATICS

JOHN BIRD

Bird's Basic Engineering Mathematics

Why is knowledge of mathematics important in engineering?

A career in any engineering or scientific field will require both basic and advanced mathematics. Without mathematics to determine principles, calculate dimensions and limits, explore variations, prove concepts, and so on, there would be no mobile telephones, televisions, stereo systems, video games, microwave ovens, computers or virtually anything electronic. There would be no bridges, tunnels, roads, skyscrapers, automobiles, ships, planes, rockets or most things mechanical. There would be no metals beyond the common ones, such as iron and copper, no plastics, no synthetics. In fact, society would most certainly be less advanced without the use of mathematics throughout the centuries and into the future.

Electrical engineers require mathematics to design, develop, test, or supervise the manufacturing and installation of electrical equipment, components or systems for commercial, industrial, military or scientific use.

Mechanical engineers require mathematics to perform engineering duties in planning and designing tools, engines, machines and other mechanically functioning equipment; they oversee installation, operation, maintenance and repair of such equipment as centralised heat, gas, water and steam systems.

Aerospace engineers require mathematics to perform a variety of engineering work in designing, constructing and testing aircraft, missiles, and spacecraft; they conduct basic and applied research to evaluate adaptability of materials and equipment to aircraft design and manufacture and recommend improvements in testing equipment and techniques.

Nuclear engineers require mathematics to conduct research on nuclear engineering problems or apply principles and theory of nuclear science to problems concerned with release, control and utilisation of nuclear energy and nuclear waste disposal.

Petroleum engineers require mathematics to devise methods to improve oil and gas well production and determine the need for new or modified tool designs; they oversee drilling and offer technical advice to achieve economical and satisfactory progress.

Industrial engineers require mathematics to design, develop, test and evaluate integrated systems for managing industrial production processes, including human work factors, quality control, inventory control, logistics and material flow, cost analysis and production coordination.

Environmental engineers require mathematics to design, plan, or perform engineering duties in the prevention, control and remediation of environmental health hazards, using various engineering disciplines; their work may include waste treatment, site remediation or pollution control technology.

Civil engineers require mathematics in all levels in civil engineering – structural engineering, hydraulics and geotechnical engineering are all fields that employ mathematical tools such as differential equations, tensor analysis, field theory, numerical methods and operations research.

Knowledge of mathematics is therefore needed by each of the engineering disciplines listed above.

It is intended that this text – *Bird's Basic Engineering Mathematics* – will provide a step by step approach to learning all the early, fundamental mathematics needed for your future engineering studies.

Now in its eighth edition, *Bird's Basic Engineering Mathematics* has helped thousands of students to succeed in their exams. Mathematical theories are explained in a straightforward manner, supported by practical engineering examples and applications to ensure that readers can relate theory to practice. Some 1,000 engineering situations/problems have been 'flagged-up' to help demonstrate that engineering cannot be fully understood without a good knowledge of mathematics.

The extensive and thorough coverage makes this a great text for introductory level engineering courses – such as for aeronautical, construction, electrical, electronic, mechanical, manufacturing engineering and vehicle technology – including for BTEC First, National and Diploma syllabuses, City & Guilds Technician Certificate and Diploma syllabuses, and even for GCSE revision.

Its companion website provides extra materials for students and lecturers, including full solutions for all 1,700 further questions, lists of essential formulae, multiple choice tests, and illustrations, as well as full solutions to revision tests for course instructors.

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To Sue

Bird's Basic Engineering Mathematics

Eighth Edition

John Bird

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Preface

Bird's Basic Engineering Mathematics 8th Edition introduces and then consolidates basic mathematical principles and promotes awareness of mathematical concepts for students needing a broad base for further vocational studies. In this eighth edition, examples and problems where ***engineering applications*** occur have been 'flagged up', new multiple-choice questions have been added to each chapter, the text has been added to and simplified, together with other minor modifications.

The text covers:

(i) **Basic mathematics** for a wide range of introductory/access/foundation mathematics courses
(ii) Mathematics contents of courses on **Engineering Principles**
(iii) '**Mathematics for Engineering Technicians**' for BTEC First NQF Level 2; ***chapters 1 to 12, 16 to 18, 21, 22, 24, and 26 to 28 are needed for this module.***
(iv) The mandatory '**Mathematics for Technicians'** for BTEC National Certificate and National Diploma in Engineering, NQF Level 3; ***chapters 7 to 10, 14 to 17, 19, 21 to 24, 26 to 28, 32, 33, 35 and 36 are needed for this module. In addition, chapters 1 to 6, 11 and 12 are helpful revision for this module.***
(v) **GCSE revision**, and for similar mathematics courses in English-speaking countries worldwide.

Bird's Basic Engineering Mathematics 8th Edition provides a lead into *Bird's Engineering Mathematics 9th Edition.*

Each topic considered in the text is presented in a way that assumes in the reader little previous knowledge of that topic.

Theory is introduced in each chapter by an outline of essential theory, definitions, formulae, laws and procedures. However, these are kept to a minimum, for problem solving is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then solving similar problems themselves.

This textbook contains over **800 worked problems**, followed by some **1700 further problems** (all with answers - at the end of the book). The further problems are contained within **201 Practice Exercises**; each Practice Exercise follows on directly from the relevant section of work. Fully worked solutions to all 1700 problems have been made freely available to all via the website www.routledge.com/cw/bird – see below. **427 line diagrams** enhance the understanding of the theory. Where at all possible the problems mirror potential practical situations found in engineering and science. In fact, some **1000 engineering situations/problems** have been 'flagged-up' to help demonstrate that engineering cannot be fully understood without a good knowledge of mathematics.

At regular intervals throughout the text are **15 Revision Tests** to check understanding. For example, Revision Test 1 covers material contained in chapters 1 and 2, Revision Test 2 covers the material contained in chapters 3 to 5, and so on. These Revision Tests do not have answers given since it is envisaged that lecturers/instructors could set the Tests for students to attempt as part of their course structure. Lecturers/instructors may obtain solutions to the Revision Tests in an **Instructor's Manual** available online at www.routledge.com/cw/bird – see below.

At the end of the book a list of relevant **formulae** contained within the text is included for convenience of reference.

'Learning by Example' is at the heart of *Bird's Basic Engineering Mathematics 8th Edition.*

JOHN BIRD
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Free Web downloads at
www.routledge.com/cw/bird

For students

1. **Full solutions** to the 1700 questions contained in the 201 Practice Exercises
2. **List of essential formulae**
3. **Famous engineers/scientists** – From time to time in the text, 18 famous mathematicians/engineers are referred to and emphasised with an asterisk*. Background information on each of these is available via the website. Mathematicians/engineers involved are: **Boyle, Celsius, Charles, Descartes, Faraday, Henry, Hertz, Hooke, Kirchhoff, Leibniz, Morland, Napier, Newton, Ohm, Pascal, Pythagoras, Simpson and Young.**

For instructors/lecturers

1. **Full solutions** to the 1700 questions contained in the 201 Practice Exercises
2. **Full solutions** and marking scheme to each of the **15 Revision Tests**
3. **Revision Tests** – available to run off to be given to students
4. **List of essential formulae**
5. **Illustrations** – all 427 available on PowerPoint
6. **Famous engineers/scientists** – 18 are mentioned in the text, as listed previously.

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The publisher wishes to thank CASIO Electronic Co. Ltd, London for permission to reproduce the image of the Casio fx-991ES calculator on page 27.

The publisher also wishes to thank the AA Media Ltd for permission to reproduce the map of Portsmouth on page 166.

Chapter 1

Basic arithmetic

***Why it is important to understand:* Basic arithmetic**

Being numerate, i.e. having an ability to add, subtract, multiply and divide whole numbers with some confidence, goes a long way towards helping you become competent at mathematics. Of course electronic calculators are a marvellous aid to the quite complicated calculations often required in engineering; however, having a feel for numbers 'in our head' can be invaluable when estimating. Do not spend too much time on this chapter because we deal with the calculator later; however, try to have some idea how to do quick calculations in the absence of a calculator. You will feel more confident in dealing with numbers and calculations if you can do this.

At the end of this chapter you should be able to:

- understand positive and negative integers
- add and subtract integers
- multiply and divide two integers
- multiply numbers up to 12×12 by rote
- determine the highest common factor from a set of numbers
- determine the lowest common multiple from a set of numbers
- appreciate the order of operation when evaluating expressions
- understand the use of brackets in expressions
- evaluate expressions containing $+, -, \times,$ and brackets

1.1 Introduction

Whole numbers

Whole Numbers are simply the numbers **0, 1, 2, 3, 4, 5,** ...

Counting numbers

Counting Numbers are whole numbers, but **without the zero**, i.e. **1, 2, 3, 4, 5,** ...

Natural numbers

Natural Numbers can mean either counting numbers or whole numbers.

Integers

Integers are like whole numbers, but they **also include negative numbers**.

Examples of integers include $\ldots -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots$

Arithmetic operators

The four basic arithmetic operators are add (+), subtract (−), multiply (×) and divide (÷).

It is assumed that adding, subtracting, multiplying and dividing reasonably small numbers can be achieved without a calculator. However, if revision of this area is needed then some worked problems are included in the following sections.

When **unlike signs** occur together in a calculation, the overall sign is **negative**. For example,

$$3 + (-4) = 3 + -4 = 3 - 4 = \mathbf{-1}$$

and

$$(+5)\times(-2) = \mathbf{-10}$$

Like signs together give an overall **positive sign**. For example,

$$3-(-4) = 3 - -4 = 3+4 = \mathbf{7}$$

and

$$(-6)\times(-4) = \mathbf{+24}$$

Prime numbers

A prime number can be divided, without a remainder, only by itself and by 1. For example, 17 can be divided only by 17 and by 1. Other examples of prime numbers are 2, 3, 5, 7, 11, 13, 19 and 23.

1.2 Revision of addition and subtraction

You can probably already add two or more numbers together and subtract one number from another. However, if you need revision then the following worked problems should be helpful.

Problem 1. Determine 735 + 167

$$\begin{array}{r} \mathbf{H\ T\ U} \\ 7\ 3\ 5 \\ +\ 1\ 6\ 7 \\ \hline 9\ 0\ 2 \\ \hline 1\ 1\ \ \end{array}$$

(i) 5 + 7 = 12. Place 2 in units (U) column. Carry 1 in the tens (T) column.

(ii) 3 + 6 + 1 (carried) = 10. Place the 0 in the tens column. Carry the 1 in the hundreds (H) column.

(iii) 7 + 1 + 1 (carried) = 9. Place the 9 in the hundreds column.

Hence, **735 + 167 = 902**

Problem 2. Determine 632 − 369

$$\begin{array}{r} \mathbf{H\ T\ U} \\ 6\ 3\ 2 \\ -\ 3\ 6\ 9 \\ \hline 2\ 6\ 3 \\ \hline \end{array}$$

(i) 2 − 9 is not possible; therefore change one ten into ten units (leaving 2 in the tens column). In the units column, this gives us 12 − 9 = 3

(ii) Place 3 in the units column.

(iii) 2 − 6 is not possible; therefore change one hundred into ten tens (leaving 5 in the hundreds column). In the tens column, this gives us 12 − 6 = 6

(iv) Place the 6 in the tens column.

(v) 5 − 3 = 2

(vi) Place the 2 in the hundreds column.

Hence, **632 − 369 = 263**

Problem 3. Add 27, −74, 81 and −19

This problem is written as 27 − 74 + 81 − 19.

Adding the positive integers:	27
	81
Sum of positive integers is	108
Adding the negative integers:	74
	19
Sum of negative integers is	93
Taking the sum of the negative integers from the sum of the positive integers gives	108
	−93
	15

Thus, **27 − 74 + 81 − 19 = 15**

Problem 4. Subtract −74 from 377

This problem is written as 377 − −74. Like signs together give an overall positive sign, hence

$$377 - -74 = 377 + 74 \qquad \begin{array}{r} 3\ 7\ 7 \\ +\ \ \ 7\ 4 \\ \hline 4\ 5\ 1 \\ \hline \end{array}$$

Thus, **377 − −74 = 451**

Problem 5. Subtract 243 from 126

The problem is 126 − 243. When the second number is larger than the first, take the smaller number from the larger and make the result negative. Thus,

$$126 - 243 = -(243 - 126) \qquad \begin{array}{r} 2\ 4\ 3 \\ -\ 1\ 2\ 6 \\ \hline 1\ 1\ 7 \\ \hline \end{array}$$

Thus, **126 − 243 = −117**

Problem 6. Subtract 318 from −269

The problem is −269 − 318. The sum of the negative integers is

$$\begin{array}{r} 269 \\ +318 \\ \hline 587 \\ \hline \end{array}$$

Thus, **−269 − 318 = −587**

Now try the following Practice Exercise

Practice Exercise 1 Further problems on addition and subtraction (answers on page 442)

In Problems 1–15, determine the values of the expressions given, without using a calculator.

1. 67 kg − 82 kg + 34 kg
2. 73 m − 57 m
3. 851 mm − 372 mm
4. 124 − 273 + 481 − 398
5. £927 − £114 + £182 − £183 − £247
6. 647 − 872
7. 2417 − 487 + 2424 − 1778 − 4712
8. −38419 − 2177 + 2440 − 799 + 2834
9. £2715 − £18250 + £11471 − £1509 + £113274
10. 47 + (−74) − (−23)
11. 813 − (−674)
12. 3151 − (−2763)
13. 4872 g − 4683 g
14. −23148 − 47724
15. $53774 − $38441
16. Calculate the diameter d and dimensions A and B for the template shown in Fig. 1.1. All dimensions are in millimetres.

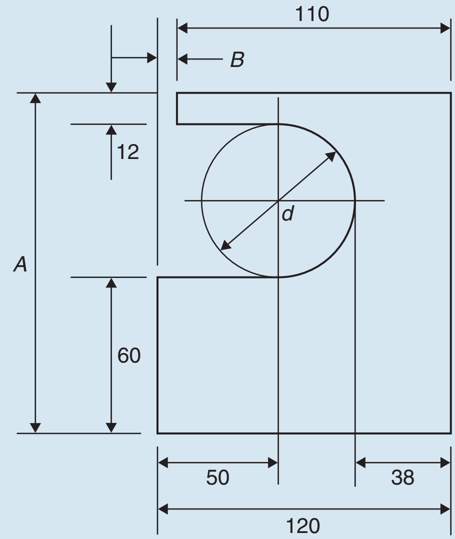

Figure 1.1

1.3 Revision of multiplication and division

You can probably already multiply two numbers together and divide one number by another. However, if you need a revision then the following worked problems should be helpful.

Problem 7. Determine 86 × 7

$$\begin{array}{rrr} \textbf{H} & \textbf{T} & \textbf{U} \\ & 8 & 6 \\ \times & & 7 \\ \hline 6 & 0 & 2 \\ \hline & 4 & \end{array}$$

(i) 7 × 6 = 42. Place the 2 in the units (U) column and ‘carry’ the 4 into the tens (T) column.

(ii) 7 × 8 = 56; 56 + 4 (carried) = 60. Place the 0 in the tens column and the 6 in the hundreds (H) column.

Hence, 86 × 7 = 602

A good grasp of **multiplication tables** is needed when multiplying such numbers; a reminder of the multiplication table up to 12 × 12 is shown below. Confidence with handling numbers will be greatly improved if this table is memorised.

Multiplication table

×	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Problem 8. Determine 764 × 38

$$\begin{array}{r} 764 \\ \times \quad 38 \\ \hline 6112 \\ 22920 \\ \hline 29032 \\ \hline \end{array}$$

(i) $8 \times 4 = 32$. Place the 2 in the units column and carry 3 into the tens column.

(ii) $8 \times 6 = 48; 48 + 3$ (carried) $= 51$. Place the 1 in the tens column and carry the 5 into the hundreds column.

(iii) $8 \times 7 = 56; 56 + 5$ (carried) $= 61$. Place 1 in the hundreds column and 6 in the thousands column.

(iv) Place 0 in the units column under the 2

(v) $3 \times 4 = 12$. Place the 2 in the tens column and carry 1 into the hundreds column.

(vi) $3 \times 6 = 18; 18 + 1$ (carried) $= 19$. Place the 9 in the hundreds column and carry the 1 into the thousands column.

(vii) $3 \times 7 = 21; 21 + 1$ (carried) $= 22$. Place 2 in the thousands column and 2 in the ten thousands column.

(viii) $6112 + 22920 = 29032$

Hence, $\mathbf{764 \times 38 = 29032}$

Again, knowing multiplication tables is rather important when multiplying such numbers.

It is appreciated, of course, that such a multiplication can, and probably will, be performed using a **calculator**. However, there are times when a calculator may not be available and it is then useful to be able to calculate the 'long way'.

Problem 9. Multiply 178 by −46

When the numbers have different signs, the result will be negative. (With this in mind, the problem can now be solved by multiplying 178 by 46.) Following the procedure of Problem 8 gives

$$\begin{array}{r} 178 \\ \times \quad 46 \\ \hline 1068 \\ 7120 \\ \hline 8188 \\ \hline \end{array}$$

Thus, $178 \times 46 = 8188$ and $\mathbf{178 \times (-46) = -8188}$

Problem 10. Determine 1834 ÷ 7

$$\begin{array}{r} 262 \\ 7\overline{)1834} \end{array}$$

(i) 7 into 18 goes 2, remainder 4. Place the 2 above the 8 of 1834 and carry the 4 remainder to the next digit on the right, making it 43

(ii) 7 into 43 goes 6, remainder 1. Place the 6 above the 3 of 1834 and carry the 1 remainder to the next digit on the right, making it 14

(iii) 7 into 14 goes 2, remainder 0. Place 2 above the 4 of 1834

Hence, $1834 \div 7 = 1834/7 = \dfrac{1834}{7} = \mathbf{262}$

The method shown is called **short division**.

Problem 11. Determine $5796 \div 12$

```
     483
 12)5796
    48
     99
     96
      36
      36
      00
```

(i) 12 into 5 won't go. 12 into 57 goes 4; place 4 above the 7 of 5796

(ii) $4 \times 12 = 48$; place the 48 below the 57 of 5796

(iii) $57 - 48 = 9$

(iv) Bring down the 9 of 5796 to give 99

(v) 12 into 99 goes 8; place 8 above the 9 of 5796

(vi) $8 \times 12 = 96$; place 96 below the 99

(vii) $99 - 96 = 3$

(viii) Bring down the 6 of 5796 to give 36

(ix) 12 into 36 goes 3 exactly.

(x) Place the 3 above the final 6

(xi) $3 \times 12 = 36$; Place the 36 below the 36

(xii) $36 - 36 = 0$

Hence, $5796 \div 12 = 5796/12 = \dfrac{5796}{12} = \mathbf{483}$

The method shown is called **long division**.

Now try the following Practice Exercise

Practice Exercise 2 Further problems on multiplication and division (answers on page 442)

Determine the values of the expressions given in Problems 1 to 9, without using a calculator.

1. (a) 78×6 (b) 124×7
2. (a) £261 × 7 (b) £462 × 9
3. (a) 783 kg × 11 (b) 73 kg × 8
4. (a) 27 mm × 13 (b) 77 mm × 12
5. (a) 448×23 (b) $143 \times (-31)$
6. (a) 288 m ÷ 6 (b) 979 m ÷ 11
7. (a) $\dfrac{1813}{7}$ (b) $\dfrac{896}{16}$
8. (a) $\dfrac{21424}{13}$ (b) $15900 \div -15$
9. (a) $\dfrac{88737}{11}$ (b) $46858 \div 14$
10. A screw has a mass of 15 grams. Calculate, in kilograms, the mass of 1200 such screws (1 kg = 1000 g).
11. A builder needs to clear a site of bricks and top soil. The total weight to be removed is 696 tonnes. Trucks can carry a maximum load of 24 tonnes. Determine the number of truck loads needed to clear the site.
12. A machine can produce 400 springs in a day. Calculate the number of springs that can be produced using 7 machines in a 5-day working week.

1.4 Highest common factors and lowest common multiples

When two or more numbers are multiplied together, the individual numbers are called **factors**. Thus, a factor is a number which divides into another number exactly. The **highest common factor (HCF)** is the largest number which divides into two or more numbers exactly.
For example, consider the numbers 12 and 15
The factors of 12 are 1, 2, 3, 4, 6 and 12 (i.e. all the numbers that divide into 12).

The factors of 15 are 1, 3, 5 and 15 (i.e. all the numbers that divide into 15).
1 and 3 are the only **common factors**; i.e. numbers which are factors of **both** 12 and 15
Hence, **the HCF of 12 and 15 is 3** since 3 is the highest number which divides into **both** 12 and 15
A **multiple** is a number which contains another number an exact number of times. The smallest number which is exactly divisible by each of two or more numbers is called the **lowest common multiple (LCM)**.
For example, the multiples of 12 are 12, 24, 36, 48, 60, 72, ... and the multiples of 15 are 15, 30, 45, 60, 75, ...
60 is a common multiple (i.e. a multiple of **both** 12 and 15) and there are no lower common multiples.
Hence, **the LCM of 12 and 15 is 60** since 60 is the lowest number that both 12 and 15 divide into.
Here are some further problems involving the determination of HCFs and LCMs.

Problem 12. Determine the HCF of the numbers 12, 30 and 42

Probably the simplest way of determining an HCF is to express each number in terms of its lowest factors. This is achieved by repeatedly dividing by the prime numbers 2, 3, 5, 7, 11, 13, . . . (where possible) in turn. Thus,

$$12 = 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$42 = 2 \times 3 \times 7$$

The factors which are common to each of the numbers are 2 in column 1 and 3 in column 3, shown by the broken lines. Hence, **the HCF is 2 × 3**; i.e. **6**. That is, 6 is the largest number which will divide into 12, 30 and 42.

Problem 13. Determine the HCF of the numbers 30, 105, 210 and 1155

Using the method shown in Problem 12:

$$30 = 2 \times 3 \times 5$$
$$105 = 3 \times 5 \times 7$$
$$210 = 2 \times 3 \times 5 \times 7$$
$$1155 = 3 \times 5 \times 7 \times 11$$

The factors which are common to each of the numbers are 3 in column 2 and 5 in column 3. Hence, **the HCF is 3 × 5 = 15**

Problem 14. Determine the LCM of the numbers 12, 42 and 90

The LCM is obtained by finding the lowest factors of each of the numbers, as shown in Problems 12 and 13 above, and then selecting the largest group of any of the factors present. Thus,

$$12 = 2 \times 2 \times 3$$
$$42 = 2 \times 3 \times 7$$
$$90 = 2 \times 3 \times 3 \times 5$$

The largest group of any of the factors present is shown by the broken lines and is 2 × 2 in 12, 3 × 3 in 90, 5 in 90 and 7 in 42
Hence, **the LCM is 2 × 2 × 3 × 3 × 5 × 7 = 1260** and is the smallest number which 12, 42 and 90 will all divide into exactly.

Problem 15. Determine the LCM of the numbers 150, 210, 735 and 1365

Using the method shown in Problem 14 above:

$$150 = 2 \times 3 \times 5 \times 5$$
$$210 = 2 \times 3 \times 5 \times 7$$
$$735 = 3 \times 5 \times 7 \times 7$$
$$1365 = 3 \times 5 \times 7 \times 13$$

Hence, **the LCM is 2 × 3 × 5 × 5 × 7 × 7 × 13 = 95550**

Now try the following Practice Exercise

Practice Exercise 3 Further problems on highest common factors and lowest common multiples (answers on page 442)

Find (a) the HCF and (b) the LCM of the following groups of numbers.

1. 8, 12
2. 60, 72
3. 50, 70
4. 270, 900
5. 6, 10, 14
6. 12, 30, 45

7.	10, 15, 70, 105	8.	90, 105, 300
9.	196, 210, 462, 910	10.	196, 350, 770

1.5 Order of operation and brackets

Order of operation

Sometimes addition, subtraction, multiplication, division, powers and brackets may all be involved in a calculation. For example,

$$5-3\times 4+24\div(3+5)-3^2$$

This is an extreme example but will demonstrate the order that is necessary when evaluating.
When we read, we read from left to right. However, with mathematics there is a definite order of precedence which we need to adhere to. The order is as follows:

Brackets
Order (or p**O**wer)
Division
Multiplication
Addition
Subtraction

Notice that the first letters of each word spell **BODMAS**, a handy aide-mémoire. **O**rder means p**O**wer. For example, $4^2=4\times 4=16$
$5-3\times 4+24\div(3+5)-3^2$ is evaluated as follows:

$5-3\times 4+24\div(3+5)-3^2$

$=5-3\times 4+24\div 8-3^2$ (**B**racket is removed and $3+5$ replaced with 8)

$=5-3\times 4+24\div 8-9$ (**O**rder means p**O**wer; in this case, $3^2=3\times 3=9$)

$=5-3\times 4+3-9$ (**D**ivision: $24\div 8=3$)

$=5-12+3-9$ (**M**ultiplication: $-3\times 4=-12$)

$=8-12-9$ (**A**ddition: $5+3=8$)

$=\mathbf{-13}$ (**S**ubtraction: $8-12-9=-13$)

In practice, **it does not matter if multiplication is performed before division or if subtraction is performed before addition**. What is important is that **the process of multiplication and division must be completed before addition and subtraction**.

Brackets and operators

The basic laws governing the **use of brackets and operators** are shown by the following examples.

(a) $2+3=3+2$; i.e. the order of numbers when adding does not matter.

(b) $2\times 3=3\times 2$; i.e. the order of numbers when multiplying does not matter.

(c) $2+(3+4)=(2+3)+4$; i.e. the use of brackets when adding does not affect the result.

(d) $2\times(3\times 4)=(2\times 3)\times 4$; i.e. the use of brackets when multiplying does not affect the result.

(e) $2\times(3+4)=2(3+4)=2\times 3+2\times 4$; i.e. a number placed outside of a bracket indicates that the whole contents of the bracket must be multiplied by that number.

(f) $(2+3)(4+5)=(5)(9)=5\times 9=45$; i.e. adjacent brackets indicate multiplication.

(g) $2[3+(4\times 5)]=2[3+20]=2\times 23=46$; i.e. when an expression contains inner and outer brackets, **the inner brackets are removed first**.

Here are some further problems in which BODMAS needs to be used.

Problem 16. Find the value of $6+4\div(5-3)$

The order of precedence of operations is remembered by the word BODMAS. Thus,

$6+4\div(5-3)=6+4\div 2$ (**B**rackets)
$=6+2$ (**D**ivision)
$=\mathbf{8}$ (**A**ddition)

Problem 17. Determine the value of $13-2\times 3+14\div(2+5)$

$13-2\times 3+14\div(2+5)=13-2\times 3+14\div 7$ (B)
$=13-2\times 3+2$ (D)
$=13-6+2$ (M)
$=15-6$ (A)
$=\mathbf{9}$ (S)

Problem 18. Evaluate
$16 \div (2+6) + 18[3 + (4 \times 6) - 21]$

$16 \div (2+6) + 18[3 + (4 \times 6) - 21]$

$= 16 \div (2+6) + 18[3 + 24 - 21]$ (B: inner bracket is determined first)

$= 16 \div 8 + 18 \times 6$ (B)

$= 2 + 18 \times 6$ (D)

$= 2 + 108$ (M)

$= \mathbf{110}$ (A)

Note that a number outside of a bracket multiplies all that is inside the brackets. In this case,

$$18[3 + 24 - 21] = 18[6], \text{ which means } 18 \times 6 = 108$$

Problem 19. Find the value of
$23 - 4(2 \times 7) + \frac{(144 \div 4)}{(14 - 8)}$

$23 - 4(2 \times 7) + \frac{(144 \div 4)}{(14 - 8)} = 23 - 4 \times 14 + \frac{36}{6}$ (B)

$= 23 - 4 \times 14 + 6$ (D)

$= 23 - 56 + 6$ (M)

$= 29 - 56$ (A)

$= \mathbf{-27}$ (S)

Problem 20. Evaluate
$\frac{3 + \sqrt{(5^2 - 3^2)} + 2^3}{1 + (4 \times 6) \div (3 \times 4)} + \frac{15 \div 3 + 2 \times 7 - 1}{3 \times \sqrt{4} + 8 - 3^2 + 1}$

$$\frac{3 + \sqrt{(5^2 - 3^2)} + 2^3}{1 + (4 \times 6) \div (3 \times 4)} + \frac{15 \div 3 + 2 \times 7 - 1}{3 \times \sqrt{4} + 8 - 3^2 + 1}$$

$$= \frac{3 + 4 + 8}{1 + 24 \div 12} + \frac{15 \div 3 + 2 \times 7 - 1}{3 \times 2 + 8 - 9 + 1}$$

$$= \frac{3 + 4 + 8}{1 + 2} + \frac{5 + 2 \times 7 - 1}{3 \times 2 + 8 - 9 + 1}$$

$$= \frac{15}{3} + \frac{5 + 14 - 1}{6 + 8 - 9 + 1}$$

$$= 5 + \frac{18}{6}$$

$$= 5 + 3 = \mathbf{8}$$

Now try the following Practice Exercise

Practice Exercise 4 Further problems on order of precedence and brackets (answers on page 442)

Evaluate the following expressions.

1. $14 + 3 \times 15$
2. $17 - 12 \div 4$
3. $86 + 24 \div (14 - 2)$
4. $7(23 - 18) \div (12 - 5)$
5. $63 - 8(14 \div 2) + 26$
6. $\frac{40}{5} - 42 \div 6 + (3 \times 7)$
7. $\frac{(50 - 14)}{3} + 7(16 - 7) - 7$
8. $\frac{(7 - 3)(1 - 6)}{4(11 - 6) \div (3 - 8)}$
9. $\frac{(3 + 9 \times 6) \div 3 - 2 \div 2}{3 \times 6 + (4 - 9) - 3^2 + 5}$
10. $\frac{(4 \times 3^2 + 24) \div 5 + 9 \times 3}{2 \times 3^2 - 15 \div 3} + \frac{2 + 27 \div 3 + 12 \div 2 - 3^2}{5 + (13 - 2 \times 5) - 4}$
11. $\frac{1 + \sqrt{25} + 3 \times 2 - 8 \div 2}{3 \times 4 - \sqrt{(3^2 + 4^2)} + 1} - \frac{(4 \times 2 + 7 \times 2) \div 11}{\sqrt{9} + 12 \div 2 - 2^3}$

Practice Exercise 5 Multiple-choice questions on basic arithmetic (answers on page 442)

Each question has only one correct answer

1. $(-5) - (-2) + (-3)$ is equal to:
   (a) -4 (b) 0 (c) -6 (d) -10
2. Which of the following numbers is not an integer?
   (a) 0 (b) 2 (c) $\frac{1}{4}$ (d) -3

3. $6 \times (-2) - 18 \div 2 - 5$ is equal to:
(a) -18 (b) -26 (c) 16 (d) -6

4. Which of the following is not a prime number?
(a) 8 (b) 7 (c) 2 (d) 11

5. $15 - 3 \times 2 + 16 \div 2 + 6$ is equal to:
(a) 23 (b) 26 (c) 38 (d) 11

6. Which prime numbers lies between 19 and 28?
(a) 20 (b) 23 (c) 25 (d) 27

7. The lowest common multiple of 15 and 18 is:
(a) 90 (b) 180 (c) 270 (d) 360

8. $45 + 30 \div (21 - 6) - 2 \times 5 + 1$ is equal to:
(a) -4 (b) 35 (c) -7 (d) 38

9. The highest common factor of 54 and 60 is:
(a) 2 (b) 3 (c) 6 (d) 12

10. $18 \div 2 + 4 - 10[4 + (5 \times 3) - 21]$ is equal to:
(a) 11 (b) 33 (c) 23 (d) -11

11. The value of $2 + 2 - 2 \times 2 \div 2$ is:
(a) 0 (b) 10 (c) 8 (d) 2

12. The H.C.F. of 8, 9 and 25 is:
(a) 8 (b) 9 (c) 25 (d) 1

13. $(-5)^2 \times 3$ is equal to:
(a) -75 (b) 15 (c) 75 (d) -15

14. The value of $3(27 - 19) \div \frac{(4+2)}{3} + (-1)$ is:
(a) 10 (b) 24 (c) 13 (d) 11

15. The ratio between the L.C.M and H.C.F. of 5, 15 and 20 is:
(a) 9:1 (b) 12:1 (c) 11:1 (d) 4:3

For fully worked solutions to each of the problems in Practice Exercises 1 to 4 in this chapter, go to the website:
www.routledge.com/cw/bird